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Stringy negative-tension branes and the second law of thermodynamics

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Abstract: Negative energy objects generally lead to instabilities and a number of other disturbing behaviors. In particular, negative energy fluxes lead to a breakdown of the classical area theorem for black hole horizons, which can lead to violations of the second law of thermodynamics. The negative energy objects that arise in string theory involve special boundary conditions which remove the perturbative instabilities. We show that they have additional special features which allow them to evade contradiction with the second law. We identify one mechanism which applies for most orientifold planes in string theory, and distinct mechanisms for the O8-plane and the AdS soliton.

Keywords: Superstrings and Heterotic Strings, M-Theory, p-branes, Black Holes.

Contents

1. Introduction

Branes with negative tension have experienced a recent surge in popularity. In part, this stems from interest in string theoretic objects called orientifolds [1]–[4] as perturbative string calculations tell us that their tension is sometimes less than zero. In addition, negative tension branes have found interesting applications in certain brane-world scenarios (e.g. in [5] and related work). Indeed, to the authors' knowledge all such scenarios that address the gauge hierarchy problem contain some form of negative tension object.

It is important to recall that negative tension branes carry negative energy both in the sense that their total energy is less than zero and in the sense that they violate the weak null energy condition [6]. Mention of such objects naturally raises the question of whether they pose a danger to one's physical theory. On the one hand, the bold physicist may point out the orientifold construction of string theories, relying on string theory's track record of producing novel pieces of physics which turn out to be surprisingly robust. Past examples of this sort include certain kinds of topology changing processes [7] and dualities between gravitating theories and non-gravitating gauge theories [8, 9].

On the other hand, a more conservative physicist may point out excellent reasons to be skeptical of negative energies.[1] After all, unbridled negative energies generate disturbing dynamical instabilities. In addition, when gravitational effects are considered,

[1] While it is understood that negative energy fluxes can arise in quantum field theory [10, 11], there is significant evidence that such fluxes face fundamental constraints [12, 13, 14] restricting them to domains where quantum mechanical effects are important and in which their effects are more benign (see e.g. [15]). In contrast, as will be discussed in detail in section 3, the negative energies mentioned above can be relevant in purely classical regimes.

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unconstrained negative energies allow the formation of traversable wormholes [16, 17], "faster than light travel" [18, 19, 20], naked singularities [21, 22], and closed timelike curves (aka time machines) [17, 23]. Most importantly for the present work, the foundation of black hole thermodynamics rests on the classical area theorem [24] which states that, *if the energy of matter is non negative*, the area of black hole horizons cannot decrease. In the presence of negative energies, the entropy associated with black holes can decrease, and the conservative physicist would suspect that this will lead to violations of the second law of thermodynamics.

Clearly then, it is important to reconcile the bold and conservative points of view and to discover what (if any) special properties particular negative energy objects may possess that render them immune to such concerns. To date, discussions in the literature have concentrated on the fact that orientifolds have "orbifold boundary conditions," meaning that the perturbative description of an orientifold involves a quotient of some smooth spacetime by a discrete $\mathbb{Z}_2$ symmetry. For an orientifold p-plane, this is a $\mathbb{Z}_2$ symmetry that leaves a $(p, 1)$ spacetime surface invariant and reflects the other directions about this surface. Note that this quotient implies that the spacetime fails to be asymptotically flat in the usual sense and that the change of boundary conditions forbids pair production of orientifolds from the vacuum.[2] Certain other important aspects of orientifold physics can be captured by working on the two-fold covering space and requiring the physics to be $\mathbb{Z}_2$ symmetric. In particular, this removes the perturbative negative energy oscillations of the p-plane. Since the $\mathbb{Z}_2$ quotient resolves at one stroke two potential instabilities of concern to our conservative physicist, it provides much encouragement to bold physicists wishing to use such objects in brane-world scenarios.

While it may be tempting to regard the creation of closed time-like curves, wormholes, and naked singularities as intriguing possibilities rather than physical inconsistencies, few physicists would be willing to compromise the second law of thermodynamics. Indeed, a famous quote from Albert Einstein [25] states his belief that thermodynamics was the most likely element of physics to survive without change — in particular, that our understanding of thermodynamics is more reliable than either that of quantum mechanics or relativity. Presumably, it is also more reliable than our understanding of string theory.

It was argued in [26] that the orbifold boundary condition alone is not sufficient to enforce the second law for negative energy branes. We should first remark that although one can in principle violate the second law using negative energy and ordinary matter by, for example, throwing negative energy into a star and thereby lowering its entropy and temperature, such concerns are in fact removed *for ordinary matter* if the negative energy is confined to an orbifold plane. The association of the negative energy with the unusual boundary conditions makes it impossible for the orbifold to be destroyed by ordinary matter and thus forbids its negative energy from thermalizing and lowering the temperature of our star.

[2] Here we refer to orientifold points and non-compact or non-contractible orientifolds. Presumably, contractible orientifold planes must have positive energy to avoid this problem since their presence is compatible with the usual boundary conditions. This is a plausible result, though we are not aware of any literature on the subject.

On the other hand, the issue becomes much more subtle when black holes are involved. Unlike normal matter, black holes absorb the energy of any object that passes inside them. They also have the capability to hide the would-be orbifold plane. The usual tool for considerations of black hole thermodynamics is the Raychaudhuri equation [6], which shows that a negative energy object crossing the horizon of a black hole in a region of negligible shear necessarily causes null rays to defocus, so that the horizon area (and thus, at first sight, the entropy) tends to decrease. While one does not typically describe objects with orbifold boundary conditions as "moving," it is straightforward to instead consider a black hole that is thrown toward our negative tension brane. The resulting collision is the same in either case, and corresponds to a symmetric collision of two black holes with a negative tension object in the covering space. The second black hole complicates the analysis somewhat but, depending on the co-dimension q of the negative tension brane, a scaling analysis shows that choosing the black holes to be either very large (for $q = 1$) or very small (for $q > 2$) compared to the gravitational length scale of the negative tension brane makes the mass of this second black hole negligible in comparison with the amount of negative energy absorbed from the brane. In such regimes, the second black hole should not interfere strongly with the argument above.

In addition to the concern that the negative energy object will cause defocusing of light rays and produce a local decrease in the horizon area, one might have the simpler concern that absorbing a negative energy object would reduce the mass of the black hole so that the final equilibrium configuration would be expected to have smaller horizon area than the initial one. This is essentially a repeat performance of our star argument above, with the notable different that the black hole should in fact be able to thermalize the negative energy of an orientifold. However, one can easily see that this concern is groundless in most cases. A negative energy object is gravitationally repulsive, so that in order to collide, the brane and black hole must be impelled toward each other with a finite kinetic energy.[3] Since the energy of any object vanishes when held static at a black hole horizon, this kinetic energy must be as large as the rest mass of the negative tension brane in order for the brane to just reach the horizon of the black hole. In general then, the collision occurs only if the kinetic energy is large enough to counterbalance the negative contribution to the mass of the final object. While there remains a concern about black hole entropy during the collision itself, one therefore expects the final equilibrium state to have larger energy and thus larger entropy than did the initial state.

When the negative tension brane has co-dimension 1 (as considered in [5, 26]), however, this argument breaks down. Such branes are gravitationally attractive. In these cases, one needs to address the question of the entropy of the final state. The O8 plane and AdS soliton provide examples of this type; they are discussed in sections 3.2 and 4.

The authors of [26] explored collisions between a black hole and a negative tension brane in a low dimensional toy model. Interestingly, they found that entropy considerations became moot as such collisions necessarily gave rise to a spacelike (big-crunch like)

[3]That this kinetic energy is necessarily positive is most easily seen by considering the negative tension brane to be unmoving and thinking of the kinetic energy as associated with the relative motion of our black hole and its image in the two-fold covering space.

singularity that engulfed the entire space and prevented any notion of a final equilibrium configuration. Nevertheless, one intuitively suspects that this non-perturbative dynamical instability is linked to the above concerns about the second law. It is interesting to note that the instability of [26] remains when the system is embedded (as is straightforward to do[4]) in a supersymmetric system of the sort considered in [27]. An investigation is currently in progress [28] of whether the instability might be an artifact of the low dimensional setting used in [26], but preliminary results for a $2+1$-dimensional negative tension brane in AdS_{3+1} indicate that the instability will survive in higher dimensions [29].

Such a dynamical instability would be as undesirable for physical models as violations of the second law. We are therefore compelled to seek an alternative resolution of the problems identified above for the stringy negative tension objects. We will see that this study uncovers a new and perhaps surprising property of (most) orientifolds that removes any concern about violations of black hole thermodynamics. This property is related to an unusual set of couplings between the gravitational field and massive modes associated with various classes of non-perturbative stringy effects. Because they involve the curvature, such couplings guarantee that black hole entropy does not reduce strictly to the horizon area [30, 31] for a black hole in the neighborhood of an orientifold. This allows the entropy of a black hole to increase even though its area decreases as the horizon encounters a negative tension brane.

For each known case of a perturbatively stable stringy negative energy object, we find some novel mechanism that protects the system from violations of the second law. In most cases, this mechanism is the one just mentioned. However, the D8-brane and the AdS soliton [32] (which has already been used in brane-world constructions [33]) constitute interesting exceptions. For each of them, we uncover a distinct mechanism for consistency with the second law. One suspects that stable consistent negative energy objects can exist only when they have similarly clever properties and a natural first conjecture is that these three examples provide a complete list. One therefore expects that while more simplified phenomenological descriptions of negative tension objects (e.g. of the sort used in [5]) can point the way to intriguing and useful new phenomena, they remain at best a rough approximation to the physics involved and when studied in detail are likely to have instabilities of the sort described in [26, 28].

We begin our analysis in section 2 with the case of the orientifold six-plane (O6-plane). This case has the advantage of a known strong coupling description as a purely gravitational soliton [34, 35, 36] (the Atiyah-Hitchin manifold [37]) in M theory. In such a manifestly classical setting it is straightforward to explore issues of black hole entropy and non-perturbative gravitational couplings in detail. As we will describe, at strong coupling the ten-dimensional description of the O6-plane necessarily involves not only the massless fields of type-IIA supergravity but also the eleven-dimensional Kaluza-Klein modes, which in ten-dimensions are described by an infinite tower of fields at integer multiples of the D0-brane mass. These fields are associated with the well known "exponential corrections" that deform the negative mass euclidean Taub-NUT manifold [38] to the smooth Atiyah-

[4]We thank Jon Bagger and Raman Sundrum for this observation.

Hitchin manifold, and which can be seen in a field theoretic treatment of a D2-brane probe near an O6-plane. We argue that there is clearly no violation of the second law in the M theory description, so curvature couplings to these non-perturbative corrections resolve the puzzle as outlined above.

We then consider other negative energy orientifolds in section 3. This section begins with a scaling analysis to determine whether one would expect these objects to be subject to similar effects. The answer is in the affirmative for most cases, and we find in particular that for the strongly coupled O5-plane they must again come from fields at a non-perturbative mass scale (in this case, the scale set by the D-string tension). We argue that such effects are associated with the onset of a Kosterlitz-Thouless transition [39] in the field theory of a D-string probe near an O5-plane. At strong coupling, the associated instanton/anti-instanton pairs are unbound and produce non-perturbative corrections to the corresponding moduli space metric.

We turn in section 3.2 to the O8-plane, which forms the single exceptional orientifold case. Instead of relying on non-minimally coupled massive fields, the O8-brane turns out to avoid difficulties through the requirement that it appear in the vicinity of positive tension D8-branes.

A final negative energy object is the ADS soliton, which we address in section 4. This can be considered either as a purely gravitational negative energy soliton or, to a certain extent, as a negative energy brane in one lower dimension. Despite the similarity to the O6-plane and its Atiyah-Hitchin description in M-theory, this negative tension brane avoids thermodynamic difficulties through a completely different effect. We will see that it is simply impossible for such a negative tension brane to cross the event horizon of a black hole! Interestingly, this effect is necessarily associated with a vanishing (Einstein frame) worldvolume metric on the lower-dimensional brane, again leading to a breakdown of a description in terms of pure Einstein-Hilbert gravity coupled to a negative tension brane. See [33], however, for how to use such solitons to address the hierarchy problem in brane-world settings.

2. The orientifold six-plane

For most orientifolds, we wish to argue that the spacetime description receives corrections which break the relationship between entropy and area for nearby black holes. In this section, we will explore in detail the case of an O6-plane[5] in the strongly-coupled IIA theory. The effective description at low energies is then eleven-dimensional supergravity compactified on a circle of radius $R_{11} \sim g\ell_s$ where g is the string coupling and ℓ_s is the string length. In this case, one expects the dominant corrections to the ten-dimensional geometry to come from the massive Kaluza-Klein modes associated with this compactification (and which become light in the strong coupling limit). The advantage of focusing on this case is that non-perturbative effects associated with R_{11} are fully encapsulated in the eleven-dimensional supergravity action. Thus, we gain a much more detailed understanding of

[5] In particular, the O6$^-$-plane in the notation of [40].

the effects of the corrections than in cases where the dominant corrections come from, say, massive string modes. Note that because adding additional orientifolds changes the boundary conditions, one cannot suppress corrections by considering an arbitrary number of coincident O6's, as one can with D-branes.

Recall that the tension of an O6-plane is

$$\tau_6 = -\frac{2}{(2\pi)^6 g {\ell_s}^7}\,, \tag{2.1}$$

so that the radius of curvature of the ten-dimensional metric is of order

$$R \sim G_{10}\tau_6 \sim g l_s\,, \tag{2.2}$$

where G_{10} is the ten-dimensional Newton's constant. Hence, R is of order R_{11}, but R is much larger than l_s, as well as the length scales $l_p^{(10)} = g^{1/4} l_s$ and $l_p^{(11)} = g^{1/3} l_s$ associated with the ten- and eleven-dimensional Planck lengths. Thus, while Kaluza-Klein corrections are relevant and an eleven-dimensional description is needed to capture the full physics, no further corrections need be considered.

Below, we first review how the eleven-dimensional geometry can be obtained by stringy methods (section 2.1) and then describe the geometry itself (section 2.2) which provides the resolution of our concerns from the eleven-dimensional perspective. We then discuss the entropy (section 2.3) and energy (section 2.4) from the ten-dimensional point of view.

2.1 D2-brane probe calculation

We now briefly review how to obtain the complete eleven-dimensional geometry from a probe D-brane calculation [35, 36, 38, 41], though we refer the reader to the original references for further details.

Consider a probe D2-brane with its worldvolume aligned parallel to two of the directions in the orientifold plane. A D2-brane in the orientifold background will carry a worldvolume field theory with SU(2) gauge invariance and $N = 4$ supersymmetry in $2 + 1$ dimensions. There are seven scalar fields which parametrize the directions orthogonal to the D2-brane in the ten-dimensional geometry which split naturally into four fields corresponding to the directions along the O6-plane (which decouple from the other fields) and three fields $\phi_i, i = 1, 2, 3$ (corresponding to the overall transverse space). These scalars transform in the adjoint of SU(2), and their potential energy is minimized when they all commute. At a generic minimum of the potential, the gauge group is broken to U(1) and, since we are in $2 + 1$ dimensions, this massless gauge field can be dualized to give an additional scalar σ. Since it arises from a U(1) gauge field, σ is periodically identified, and σ in fact corresponds to the M-theory circle. We therefore have a four-dimensional Coulomb branch for this field theory for which the metric on moduli space (cross the seven flat directions along the O6-plane) is just the ten-dimensional spatial geometry of the O6-plane.[6]

[6]Here we assume that the three-form potential vanishes and that the eleven-dimensional geometry is of the form $\mathbb{R} \times \mathcal{M}_{10}$ (a direct product of metrics) where $\mathbb{R}$ is the time direction. This assumption will be justified a posteriori by the fact that the result satisfies the eleven-dimensional supergravity equations of motion.

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At the classical field theory level, the metric on the space of ϕ_i, σ is just the flat metric on $(\mathbb{R}^3 \times S^1)/\mathbb{Z}_2$, where the $\mathbb{Z}_2$ identification arises from the action of the Weyl group of SU(2). Supersymmetry guarantees that the moduli space cannot be lifted by quantum corrections and that the metric on moduli space remains hyper-Kähler. By studying the action of the symmetries one can show that the one-loop contribution generates a cross term between the ϕ_i and σ. In terms of the ten-dimensional string theory, this correction corresponds to the Ramond-Ramond charge carried by the O6-plane. The hyper-Kähler structure implies that there are no new corrections at higher orders in perturbation theory, but there may be non-perturbative corrections. Indeed, one can identify instantons associated with D0-brane exchange between the probe and the orientifold that yield an infinite series of corrections exponentially suppressed by powers of the D0-brane mass. Since there is no Higgs branch for this theory, the metric on the Coulomb branch must be non singular [36]; together with the supersymmetry and asymptotic structure, this is enough to determine the metric uniquely. The appropriate metric is the Atiyah-Hitchin metric of [37]. Thus, the flat metric receives a one-loop correction which accounts for the effects of the bulk Ramond-Ramond gauge field, and instanton corrections which remove the singularity at the origin to give the Atiyah-Hitchin metric. The eleven-dimensional metric for the O6-plane is then this metric cross a flat $\mathbb{R}^{6,1}$.

2.2 The Atiyah-Hitchin metric

The Atiyah-Hitchin metric is a non-singular SO(3) symmetric hyper-Kähler manifold, which also appears in the study of a two-monopole moduli space. The metric takes the form [42]

$$ds^2 = \frac{b^2}{r^2}dr^2 + a^2\sigma_1^2 + b^2\sigma_2^2 + c^2\sigma_3^2\,, \tag{2.3}$$

where

$$\begin{aligned}
\sigma_1 &= -\sin\psi d\theta + \cos\psi\sin\theta d\phi\,,\\
\sigma_2 &= \cos\psi d\theta + \sin\psi\sin\theta d\phi\,,\\
\sigma_3 &= d\psi + \cos\theta d\phi\,,
\end{aligned} \tag{2.4}$$

and a, b, c are functions only of r. The ranges of the coordinates are $0 \le \theta \le \pi$, $0 \le \phi \le 2\pi$, $0 \le \psi \le 2\pi$, and $\pi R_{11}/2 \le r < \infty$. In addition, an identification is imposed under the $\mathbb{Z}_2$ symmetry

$$\theta \longrightarrow \pi - \theta\,, \qquad \phi \longrightarrow \phi + \pi\,, \qquad \psi \longrightarrow -\psi\,. \tag{2.5}$$

At large r, the functions a, b, c take the form

$$\begin{aligned}
a &= r\left(1 - \frac{R_{11}}{r}\right)^{1/2} - \frac{8r^2}{R_{11}}\left(1 - \frac{R_{11}^2}{8r^2}\right)e^{-2r/R_{11}} + \mathcal{O}\left(e^{-4r/R_{11}}\right),\\
b &= r\left(1 - \frac{R_{11}}{r}\right)^{1/2} + \frac{8r^2}{R_{11}}\left(1 + \frac{R_{11}}{r} - \frac{R_{11}^2}{8r^2}\right)e^{-2r/R_{11}} + \mathcal{O}\left(e^{-4r/R_{11}}\right),\\
c &= -R_{11}\left(1 - \frac{R_{11}}{r}\right)^{1/2} + \mathcal{O}\left(e^{-2r/R_{11}}\right),
\end{aligned} \tag{2.6}$$

where $2\pi R_{11}$ is the asymptotic circumference of the S^1. The above two features together imply that the geometry asymptotically approaches the flat metric on $(\mathbb{R}^3 \times S^1)/\mathbb{Z}_2$, with ψ the coordinate on the S^1. Thus, it is the angle ψ that corresponds to the M-theory circle and the reduction on this circle to ten dimensions gives a magnetic charge under the Kaluza-Klein gauge field.

The exponentially small corrections at large r represent the effect of the instantons described in the preceding section. Because of these corrections, the metric closes off in a regular fashion at $r = \pi R_{11}/2$, where $b = -c = \pi$, and $a \to 2r - \pi R_{11}$. There is no region with $r < \pi R_{11}/2$. For more details, see [42].

For our purposes, the important point is that the eleven-dimensional geometry is a completely smooth vacuum solution; the above metric is Ricci flat. Despite the vacuum geometry, this spacetime actually has a negative ADM mass (tension). The failure of the positive mass theorems [43, 44] is associated with the unusual boundary conditions which, for example, rule out the existence of the asymptotically constant spinors used in the Witten proof [44]. The complete description of the collision of a black hole with the O6-plane is then given by a black string wrapped on the compact direction in 11d "moving in the Atiyah-Hitchin space".[7] Since this spacetime satisfies the weak energy condition — indeed, there is no local stress-energy of any kind — in the 11d description, the usual eleven-dimensional area theorem tells us that the area, and hence entropy, of this black string solution is non decreasing. Thus, once we take the exponential corrections fully into account, the apparent violation of the second law is completely resolved.

2.3 Entropy from the ten-dimensional point of view

The eleven-dimensional picture removes concern about the second law and provides a very satisfying result. However, it is useful to explore the mechanism by which the second law is saved from the ten-dimensional point of view. To flesh out the details, we now re-examine the Atiyah-Hitchin metric from the ten-dimensional point of view and see how the naïve argument in the introduction is invalidated.

First of all, we note that the Atiyah-Hitchin metric (2.3) depends on ψ, so that ∂_ψ is not a Killing vector field. This implies that the dimensional reduction to ten dimensions will contain excited massive fields from the Kaluza-Klein modes. We wish to argue that the presence of these massive modes implies that the usual relation between the entropy and the area of the black hole horizon is modified. The essential point is that the formula $S = A/4$ is valid only in Einstein gravity minimally coupled to matter fields. Wald and Iyer undertook a detailed study of a family of actions with further curvature terms, and showed that the first law of black hole thermodynamics requires the definition of the entropy to be suitably generalized [30] (see also [31] for related work). The full entropy expression is obtained by differentiating the lagrangian with respect to its dependence on the Riemann tensor, so that each curvature coupling provides a separate contribution to the entropy of any black hole.

[7] Such language is strictly appropriate only for a perturbatively small black string, but it is convenient to use this phrase to refer to large black strings as well.

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In our case, the presence of the massive modes will produce such additional curvature terms. In general, the ten-dimensional action involving these massive modes is a complicated non-linear expression. However, these couplings become tractable when the massive modes can be treated as a small perturbation. This will be enough for our purposes as we seek only a qualitative understanding in ten-dimensional terms.

Writing the 11d metric as $g_{ab} = g^0_{ab} + h_{ab}$, the quadratic action for the perturbation h_{ab} is [45]

$$S = \frac{1}{32\pi G}\int d^{11}x\sqrt{-g^0}\; h^{ab}A_{abcd}h^{cd}\,, \tag{2.7}$$

where g^0_{ab} has been used to raise the indices and

$$\begin{aligned} A_{abcd} = {}& \frac{1}{4}g^0_{cd}\nabla_a\nabla_b - \frac{1}{4}g^0_{ac}\nabla_d\nabla_b + \frac{1}{8}(g^0_{ac}g^0_{bd} + g^0_{ab}g^0_{cd})\nabla_e\nabla^e + \frac{1}{2}R^0_{ad}g^0_{bc} - \frac{1}{4}R^0_{ab}g_{cd} + \\ & + \frac{1}{16}R^0 g^0_{ab}g^0_{cd} - \frac{1}{8}R^0 g^0_{ac}g^0_{bd} + (a\leftrightarrow b) + (c\leftrightarrow d) + (a\leftrightarrow b, c\leftrightarrow d)\,. \end{aligned} \tag{2.8}$$

Here R^0_{ab} (R^0) denotes the Ricci tensor (scalar) of the background spacetime g^0_{ab}. Similarly, ∇ is the covariant derivative associated with g^0_{ab}. We are mostly interested in the curvature couplings, which can be rewritten as

$$S_{\rm curv} = \frac{1}{32\pi G}\int d^{11}x\sqrt{-g^0}R^0_{ab}\left(2h^{ac}h_c{}^b - h^{ab}h_c{}^c + \frac{1}{4}h_c{}^c h_d{}^d - \frac{1}{2}h^{ab}h_{ab}\right). \tag{2.9}$$

We take the background g^0_{ab} to be independent of ψ and expand h_{ab} in Fourier modes,

$$h_{ab} = \sum_{m\neq 0} h^{(m)}_{ab}e^{im\psi}\,. \tag{2.10}$$

Note that we have taken the perturbation of the zero-mode to vanish, as the perturbation is to represent only the massive fields. It is for this reason that terms in the action that are linear in h_{ab} integrate to zero regardless of whether g^0_{ab} is taken to be a solution of the equations of motion. Reduction of (2.9) to ten-dimensions is straightforward once a few conventions are fixed. We choose to work with what we call the "naïve Einstein metric" $g^{10,E}$ (and the associated dilaton ϕ and 1-form A_a) in ten-dimensions which is given by the usual algebraic expressions in terms of g^0_{ab}:

$$g^0_{ab} = e^{-\phi/6}g^{E,10}_{ij}dx^i dx^j + e^{4\phi/3}(d\psi + A_i dx^i)^2\,. \tag{2.11}$$

Here we use i, j to represent ten-dimensional indices. We use the phrase "naïve Einstein metric" because the associated action is readily seen to contain the curvature couplings

$$\begin{aligned} S_{\rm curv} \sim {}& \int d^{10}x e^{-\phi/6}\sqrt{-g^{E,10}}R^{(E,10)}_{ij}x^i_{,a}x^j_{,b}\,\times \\ & \times\sum_m\left(2h^{ac(m)}h_c{}^{b(m)} - h^{ab(m)}h_c{}^{c(m)} + \frac{1}{4}h_c{}^{c(m)}h_d{}^{d(m)} - \frac{1}{2}h^{ab(m)}h^{(m)}_{ab}\right), \end{aligned} \tag{2.12}$$

where $R^{(E,10)}_{ij}$ is the Ricci tensor of $g^{E,10}_{ij}$ and the matrix of derivatives $x^i_{,a}$ implements the pull back of ten-dimensional forms to eleven dimensions.

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While this calculation is only valid when the massive modes are a small perturbation, we wish to suggest that similar considerations apply when the massive modes are of order one, as in the region near $r = \pi R_{11}/2$ in the Atiyah-Hitchin metric. In particular, we suggest that the terms in (2.7) and the corresponding higher corrections will continue to source important curvature terms in the ten-dimensional action, which will produce order-one corrections to the entropy formula. The full entropy must after all give just the eleven-dimensional area of the black hole and there is no reason in the presence of massive excitations for this to agree with the area as measured in ten-dimensions.

Let us therefore summarize our picture of the collision of a black p-brane and an orientifold plane. When the black p-brane is absorbing the orientifold plane, massive Kaluza-Klein modes will be excited near the horizon. What happens to these modes subsequently? If the black p-brane completely engulfs the orientifold plane, we would not expect the black p-brane to be capable of supporting "massive hair". In the linearized analysis, it is easy to see that the massive modes must die off exponentially, so long as the black p-brane has a radius r_0 satisfying $r_0 \gg R_{11}$ (so that it is a truly ten-dimensional solution). This is because in eleven dimensions we are considering just perturbations of the black string of the form $h_{ab} \sim e^{im\psi}$. But these are the perturbations considered by Gregory and Laflamme [46], who showed that they will decay exponentially in time for all $m > m_* \sim R_{11}/r_0$. Thus, the massive modes are absorbed by the black p-brane, which settles down to a regular solution carrying just the magnetic Kaluza-Klein gauge charge.

From the ten-dimensional point of view, the picture we want to suggest is thus that the orientifold will first fall across the event horizon, lowering the area of the black p-brane. However, the massive Kaluza-Klein fields will then have order-one excitations in the vicinity of the horizon, so the area will differ substantially from the entropy. These massive modes will then be absorbed by the black p-brane. Presumably, this influx carries positive energy into the black brane and raises its area so that the final state will be a black brane with energy and horizon area greater than the initial one. Since the massive modes die away, the ten- and eleven-dimensional areas must once again agree in the final state. One can then see that the final area is greater from the eleven-dimensional picture where there is no negative energy. Alternatively, one can use the general argument in the introduction that the kinetic energy must be at least big enough to overcome the negative energy from the orientifold tension.

2.4 Energy from the ten-dimensional point of view

We have argued that the description of the horizon dynamics should be rather different from the ten- and eleven-dimensional points of view, with the ten-dimensional horizon shrinking during the collision while the eleven-dimensional horizon grows. We then suggested that this difference can be traced to the massive modes. In section 2.3 above we showed that these massive modes do induce a series of perturbative corrections to black hole entropy. We now verify the flip-side of our suggestion by showing that the massive modes lead to perturbative negative energy fluxes from the ten-dimensional point of view and so can be associated directly with the defocusing of null rays and the shrinking of black hole horizons. In particular, we argue that at finite coupling the negative energy is not confined to an infinitely thin brane, but instead spreads out over a finite region of space. We again study these excitations in the perturbative regime where there is a clear distinction between the massive fields and the massless IIA fields. For the solution (2.3), this corresponds to the region $r \gg R_{11}$.

We wish to show that Kaluza-Klein reduction of (2.3) to ten-dimensions yields massive modes which violate the weak null energy condition ($k^a k^b T_{ab} > 0$) for some null vectors k. Here, we speak of the stress-energy tensor associated with the naïve Einstein metric $g^{E,10}$ introduced above.

If the massive fields were the only sources in ten-dimensions, the sign of the massive mode energy would be related by the Einstein equations to the sign of $R^{E,10}_{ab} k^a k^b$, which must be positive to prove the usual black hole area theorem. In the present setting, the massless ten-dimensional matter fields (the dilaton and the Ramond-Ramond 1-form) are also excited and, in fact, dominate over the massive modes at large r. The stress-energy tensor of these massless fields satisfies the weak energy condition, and therefore so must the full stress-energy tensor in any perturbative treatment. However, our goal here is to analyze the contribution of the massive modes T^{massive}_{ij} in particular, which can be obtained by simply subtracting the massless contribution T^{massless}_{ij} from the full stress-energy tensor T^{full}_{ij}. That this difference has a simple representation in terms of eleven-dimensional quantities is readily seen by using the ten-dimensional Einstein equations (which include the effects of the massive modes) to express T^{massive}_{ij} in terms of a variation of the massless IIA action:

$$\begin{aligned} k^i k^j T^{\text{massive}}_{ij} &= k^i k^j T^{\text{full}}_{ij} - k^i k^j T^{\text{massless}}_{ij} = \frac{1}{8\pi G_{10}} k^i k^j R^{(E,10)}_{ij} - k^i k^j T^{\text{massless}}_{ij} \\ &= \frac{2}{\sqrt{-g^{E,10}}} k^i k^j \frac{\delta}{\delta g^{(10,E)ij}} S_{\text{massless IIA}} \\ &= \int d\psi \frac{e^{-\phi/6}\sqrt{-g^{11}}}{16\pi^2 G_{10}\sqrt{-g^{10,E}}} k^i k^j R^0_{ij} \,, \end{aligned} \tag{2.13}$$

where we have assumed that k is null with respect to both the ten- and eleven-dimensional metrics ($g^{E,10}_{ab}$ and g^0_{ab}) and the symbol R^0_{ab} refers to the Ricci tensor of the eleven-dimensional zero-mode metric g^0_{ab}.

As a result, we need only analyze the sign of $\int d\psi k^a k^b R^0_{ab}$ for an appropriate null vector k. Note that although the full eleven-dimensional metric $g_{ab} = g^0_{ab} + h_{ab}$ is Ricci flat in the Atiyah-Hitchin case, the zero mode piece g^0_{ab} need not be and $k^a k^b R^0_{ab}$ need not vanish. The setting here is similar to that often used to treat gravitational radiation in which we separate out a gravitational perturbation h_{ab} and choose not to regard it not as part of the metric (represented by g^0_{ab}) but instead as a "matter field" which in turn acts as a gravitational source.

Nonetheless, the vanishing of the full Ricci tensor $R^{(11)}_{ab}$ is still quite useful. In particular, we can expand $R^{(11)}_{ab}$ as

$$0 = R^{(11)}_{ab} = R^0_{ab} + R^{0,1}_{ab} + R^{0,2}_{ab} + \cdots \,, \tag{2.14}$$

where $R^{0,1}_{ab}$ and $R^{0,2}_{ab}$ are, respectively, linear and quadratic in the perturbation h^{ab}. Since h^{bc} has no zero-mode part, the integral of $R^{0,1}_{ab}$ over ψ vanishes and to second order in the perturbation we have

$$\int d\psi k^a k^b R^0_{ab} = -\int d\psi k^a k^b R^{0,2}_{ab}\,. \tag{2.15}$$

We may then consult [6] to find[8] the relation

$$\begin{aligned} R^{0,2}_{ab} = &\frac{1}{2}h^{cd}\nabla_{(a}\nabla_{b)}h_{cd} - \frac{1}{2}h^{cd}\nabla_a\nabla_c h_{bd} - \frac{1}{2}h^{cd}\nabla_b\nabla_c h_{ad} + \frac{1}{4}(\nabla_a h_{cd})\nabla_b h^{cd} + \\ &+ (\nabla^d h^c_b)\nabla_{[d}h_{c]a} - \frac{1}{2}\nabla_d(h^{dc}\nabla_c h_{ab}) - \frac{1}{4}(\nabla^c h)\nabla_c h_{ab} - \\ &- \left(\nabla_d h^{cd} - \frac{1}{2}\nabla^c h\right)\nabla_{(a}h_{bc)}\,, \end{aligned} \tag{2.16}$$

where we have used the fact that terms involving R^0_{ab} are higher order in h_{ab}. Here indices are raised and lowered with the zero-mode metric g^0_{ab} and $h = g^{0,ab}h_{ab}$ is the trace of the perturbation in this metric.

It is now straightforward to evaluate $k^a k^b R^{0,2}_{ab}$ for various null vectors k using the large r expansion of (2.3). We choose $k = k^t\frac{\partial}{\partial t} \pm k^r\frac{\partial}{\partial r}$, which is clearly relevant to the horizon of an approaching black hole. The reader may check that such k are null simultaneously for both $g^{E,10}_{ab}$ and g^0_{ab}. If we introduce

$$\Delta^2 \equiv b^2 - a^2 = \frac{2^7 r^3}{R^3_{11}}e^{-2r/R_{11}}\left(1 + \mathcal{O}\left(\frac{r}{R_{11}}\right)\right), \tag{2.17}$$

a somewhat tedious calculation yields

$$\frac{1}{2\pi}\int d\psi k^a k^b R^0_{ab} = -(k^r)^2\frac{3R^2_{11}}{32r^4}\Delta^4\left(1 + \mathcal{O}\left(\frac{r}{R_{11}}\right)\right) < 0\,. \tag{2.18}$$

As a result, the stress-energy tensor of the massive modes does indeed violate the weak energy condition in our naïve ten-dimensional Einstein frame.

Of course, this does not mean that an approaching black hole begins to contract while the orientifold is still far away. The energy in the large r regime is is dominated by the massless modes and must therefore be positive. However, we expect the massive modes to dominate once $r \sim R_{11}$, and there is no reason to expect the sign of the energy carried by such modes to change at that point. Thus, the calculation above indicates that at finite coupling the full stress-energy violates the weak energy condition over a region of size R_{11}. When the black hole encounters this region, its horizon will begin to contract.

3. Other orientifold-planes

We now want to consider the other Orientifold-planes of type-II string theory. These may be classified in much the same manner as the familiar positive tension branes in terms of the bulk gauge fields under which they carry charge. The various categories include the Op-planes for $p = 0, 1, \ldots, 8$ (which have the same supersymmetries and the same kinds of

[8]The reference [6] contains the result for perturbations around Minkowski space. However, variations of the Ricci tensor can be written in terms of only the metric perturbation h_{ab}, its covariant derivatives, and the unperturbed Ricci tensor. Thus, since terms involving R^0_{ab} and two h's are higher order, $R^{0,2}_{ab}$ can be obtained from the flat space result by simply replacing partial derivatives with covariant derivatives and checking the ordering of the second covariant derivatives against (2.8) above.

charges as the Dp-branes), the ONS5- and OF1-planes which couple to the Neveu-Schwarz B-field, and the OP1 orientifold line which is not charged under a gauge field but does carry momentum.

For the strongly coupled O6-plane, corrections from massive Kaluza-Klein modes were important because the curvature radius R of the ten-dimensional metric (the "orientifold scale") was of the same order as R_{11} while other corrections (e.g. α' corrections) could be ignored (since e.g. $R \gg \ell_s$). To discover which corrections may be important for the other O-planes, we should similarly compare their geometric scales to the fundamental length scales.

In the IIA cases, these fundamental scales are the string length ℓ_s, the 10-d Planck length $\ell_p^{(10)}$, and the scales of M theory: R_{11} and the 11-d Planck length $\ell_p^{(11)}$. While R_{11} has no direct analogue in the IIB theory, distance scales associated with the D-brane tensions lead to similar effects. The fundamental scales are related through:

$$\ell_s = g^{-1/4}\ell_p^{(10)} = g^{-1/3}\ell_p\,, \qquad \ell_p^{(10)} = \left(\ell_p^{(10)}\right)^{9/8} R_{11}^{-1/8}\,, \qquad R_{11} = g\ell_s = g^{2/3}\ell_p^{(11)}\,. \tag{3.1}$$

For the (Ramond-Ramond) Op-planes, the scale is set by the D-brane tension

$$T \sim \frac{m}{L^p} \sim \frac{1}{g\ell_s^{p+1}}\,. \tag{3.2}$$

As a result, the radius of curvature R of the 10-d metric for such cases is

$$R^{7-p} \sim G_{10}T \sim g\ell_s^{7-p}\,. \tag{3.3}$$

Consider first weak coupling. Then the string length is the largest scale, and this is at least as big as the orientifold scale except for $p = 8$. Thus for $p \neq 8$, stringy corrections can invalidate the pure supergravity description. Since these orientifolds preserve half the supersymmetry, their interactions may be protected from perturbative corrections. However, we saw explicitly that non-perturbative corrections arise in the O6 case. Dualities then imply that some corrections should be present for the other Op-planes as well. The O8-plane is an exception to this general picture; it can have no significant corrections at weak coupling and requires some new mechanism to protect the second law. The resolution of this case is discussed in section 3.2 below.

At strong coupling, the situation is reversed and $R/\ell_s \sim g^{1/(7-p)}$ is large for $p < 7$, so that $\mathrm{O}(\alpha')$ corrections are negligible for these cases. We now need to consider the other fundamental scales, which are larger than the string scale at strong coupling. We can rewrite (3.3) as

$$R^{7-p} \sim g^{(p-3)/4}[\ell_p^{(10)}]^{7-p}\,, \tag{3.4}$$

so that $R/\ell_p^{(10)}$ is also large at strong coupling and the associated corrections are small for $p > 3$. In terms of the 11-d Planck length we have

$$R^{7-p} \sim g^{(p-4)/3}\ell_p^{7-p}\,, \tag{3.5}$$

so that 11-d quantum corrections are important for $p = 4$. Thus, the only cases where quantum corrections are not important are $p = 6$, considered in the previous section, and $p = 5$. We will discuss $p = 5$ in section 3.1 below, and argue that it receives non-perturbative corrections from D-string effects that are similar to those described in section 2 for the orientifold 6-plane.

There are also a few NS-type O-planes to consider. For the ONS5, $G_{10}T \sim \ell_s^2$, and string corrections are always important. For the OF1, we have $G_{10}T \sim g^2\ell_s^6 \sim g^{1/2}\ell_p^{(10)6} \sim \ell_{pl}^6$. Thus, in the IIA theory, 11-d Planck corrections are always important while in IIB string corrections are important at weak coupling. At strong coupling in the IIB theory, the D-string scale $\ell_{D1} = g^{1/2}\ell_s$ (so that $T_{D1} \sim \ell_{D1}^{-2}$) becomes large enough that we have

$$G_{10}T_{OF1} \sim g^2\ell_s^6 = g^{-1}\ell_{D1}^6 \ll \ell_{D1}^6\,, \tag{3.6}$$

so that D-string corrections will be important. The T-dual OP1 has a fixed line along a compact direction of radius $R_{\rm compact}$, and a tension $G_{10}T \sim g^2\ell_s^8/R_{\rm compact}^2$, which is smaller than the OF1 tension for $R_{\rm compact} > \ell_s$. As a result, either the same corrections are important as in the OF1 case above or $R_{\rm compact} \leq \ell_s$ and string scale corrections must again be considered.

Thus, a scaling analysis suggests that for any orientifold but the O8-plane the space-time description would receive corrections analogous to the discussion in section 2 for the orientifold 6-plane. For the O5-plane, the only relevant corrections should arise at the scale of the D-string tension and we should be able to find them by instanton methods in a probe brane calculation analogous to that of section 2.1. We do so in section 3.1 below and then proceed to analyze the case of the O8-plane in section 3.2.

3.1 Non-perturbative corrections and the O5-plane

Let us now turn to the strongly coupled O5-plane. Our discussion below should apply to any of the variants of the O5-plane, as they differ only by discrete Neveu-Schwarz and Ramond-Ramond fluxes. While string and (ten-dimensional) Planck scale corrections are small, the effective description of strongly-coupled IIB is in fact the S-dual weakly-coupled IIB. Recall that there were no uncorrected IIB orientifolds at weak coupling. In fact, the Ramond-Ramond Op-plane is S-dual to the ONS5-plane, which had the potential for large stringy corrections at weak coupling. From the original strongly-coupled point of view, these should arise somehow from the D-strings which are becoming light. This shows that such corrections are allowed by a scaling analysis and our discussion of the second law predicts that they will be present despite the supersymmetry of the situation.

How do we verify the existence of these corrections? We recall that the non-perturbative corrections for the O6-plane can be seen in the worldvolume field theory of probe branes. A test D2-plane moving in the O6 background is described by an SU(2) gauge theory on the D2, and the Atiyah-Hitchin metric is the associated moduli space metric. The massive Kaluza-Klein excitations in the Atiyah-Hitchin metric are related to instanton corrections to the corresponding moduli space.

We now attempt to repeat this analysis for the D1/O5-system. The relevant theory is now SU(2) gauge theory in 2 euclidean dimensions. Since the topology of infinity is S^1, the space of euclidean solutions has instanton sectors associated with the winding of

U(1) subgroups around this S^1. The lowest action configuration in this sector should be a smooth euclidean solution. Indeed, a massive version of the theory broken to a U(1) subgroup is the abelian Higgs model whose vortex instantons are well-known.

In two euclidean dimensions, the action of such vortex instantons is in general logarithmically divergent. Because our fields are massless, we expect this to be the case here. Nonetheless, such instantons can still contribute to the partition function. The point is that the action of an instanton/anti-instanton pair will be finite, as the separation s of the pair will cut off the logarithmic divergence. Such pairs therefore have an action of size $S \sim \frac{1}{g} \ln s$, where we have explicitly indicated the fact that the action of any D-brane solution is proportional to $1/g$. On the other hand, the volume of phase space available for a pair with separation s is proportional to the circumference of a circle with radius s. As a result, such instanton/anti-instanton pairs have an entropy that diverges as $\ln s$, without an the accompanying factor of $1/g$. Note that at large s the pair will provide an approximate solution to the euclidean equations of motion. It is therefore clear that, for large enough g, instanton/anti-instanton pairs with large s will provide important instanton contributions to the partition function. What we have just described is of course just the usual Kosterlitz-Thouless phase transition [39] of two-dimensional euclidean systems.

This analysis verifies our prediction and shows that the moduli space for D-strings moving in the O5-background receives corrections from instantons. This result and the analogy with the O6 system strongly suggests that we think of massive fields at scales set by the D1 tension as being excited near an O5-plane. Presumably, these fields carry negative energy and affect the entropy of black holes in much the same way as in the O6 case studied in section 2.

3.2 The O8-plane

Based on the O6-case, we generally expected non-trivial corrections to the (10d) supergravity description of the orientifolds. This is borne out, with the sole exception of the O8-plane at weak coupling. For this case, all of the corrections considered above are negligible. Furthermore, as it has co-dimension one, the O8-plane will attract a positive-mass black hole, so this case is also an exception to the argument in the introduction about kinetic energy being required to impel a black hole toward a negative tension brane. Since all the corrections are suppressed, the resolution of these puzzles must lie within classical (massive) type-IIA supergravity.

Let us consider first the question of what happens when an event horizon encounters an O8-brane. We would like to suggest that the resolution of our thermodynamic concerns is related to a well-known difficulty in dealing with O8-planes and the related D8-branes. The solution corresponding to any of these objects in isolation has a singularity at a finite distance of order $\frac{1}{g}\ell_s$ from the brane. The usual interpretation of this singularity is that it is impossible to separate an O8- or D8-brane from an oppositely charged such brane by more than a distance of order $\frac{1}{g}\ell_s$. Thus, when we consider an O8-plane, we should remember that there must necessarily be nearby either an anti O8-plane (in which case the entire universe must be small as these are world-ending branes) or a D8-brane (which has positive tension). As a result, there is simply no setting where one may ask about a black

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hole (or in fact a black p-brane) which begins farther away from the O8-plane than the O8 curvature radius $R \sim \frac{1}{g}\ell_s$ and collides with the negative tension orientifold without at the same time encountering a similar amount of positive energy material.

It is instructive to briefly consider black p-branes that begin close to the orientifold. Such a black hole will act much like its cousin in empty space if the size $R_{\rm bb}$ of the black p-brane is much smaller than the O8-plane length scale $R_{\rm O8} = \frac{1}{g}\ell_s$. If $R_{\rm bb}/R_{\rm O8}$ is non negligible, we expect that distortions of this order will be present in the black brane horizon and consequently that the area A of such a black brane will differ from the area A_0 of its flat space cousin by an amount of order $A_0\frac{R_{\rm bb}}{R_{\rm O8}}$. Furthermore, the exact distortion and the corresponding effect on the horizon area will depend on the precise location of the black brane, so that the horizon area of a black brane colliding with the O8-plane will change during the collision by an amount of this order in a manner that is beyond the scope of this work to predict.

Let us also estimate the magnitude of the supposed entropy reduction due to the collision of a black p-brane with the O8-brane. The amount of negative tension absorbed by the brane is roughly proportional to the tension $T_{\rm O8}$ of the orientifold times the cross sectional area $R_{\rm bb}^{8-p}$ of the black brane, whereas the original tension of the black brane is of size $R_{\rm bb}^{7-p}/G_{10}$, where G_{10} is the ten-dimensional Newton's constant. Since $G_{10}T_{\rm O8} \sim 1/R_{\rm O8}$, we have a fractional change in tension of order $\Delta T/T \sim R_{\rm bb}/R_{\rm O8}$ and we would expect a fractional decrease in its area of the same order. But this is of the same size as the dynamical distortion term caused by the proximity of the O8-plane! These finite-distance corrections can therefore resolve the potential decrease in area. We suggest that this is exactly what happens, though it is difficult to study the effect in detail.

In the introduction, we argued that the total mass of the black hole typically could not be reduced by collision with a negative-energy source due to the repulsive gravitational potential. In the present case, this argument breaks down, because the O8-brane is of co-dimension one. However, this problem contains its own solution: since the O8-brane is co-dimension one while black branes always have co-dimension greater than one, a black p-brane cannot engulf the entire O8-brane. Instead, part of the O8 necessarily remains in the spacetime to source significant distortions of the black p-brane. Hence, although the mass of the black hole may decrease, one cannot conclude that the black hole's area decreases when it collides with the O8-brane. Note also that the above scaling analysis suggests that a black hole cannot engulf enough O8 negative tension to bring its total mass to zero or below.

4. AdS soliton

A final negative-energy object in string theory is the AdS soliton [32]. Like the Atiyah-Hitchin metric, the AdS soliton is a pure gravitational solution and is not particularly stringy in and of itself. However, it is present in string theory and its construction was motivated by stringy considerations. As a result, this object is as connected with the consistency of string theory and the second law as the objects that we have previously discussed.

The AdS soliton is an asymptotically AdS solution of the vacuum Einstein equations with a cosmological constant and arises in string theory in the context of sphere compactifications. The metric for the AdS soliton in $p+2$ dimensions is [32]

$$ds^2 = \frac{r^2}{l^2}\left[\left(1-\frac{r_0^{p+1}}{r^{p+1}}\right)d\tau^2 + (dx^i)^2 - dt^2\right] + \left(1-\frac{r_0^{p+1}}{r^{p+1}}\right)^{-1}\frac{l^2}{r^2}dr^2\,, \tag{4.1}$$

where $i = 1, \ldots, p-1$, l is the cosmological length scale, and the coordinate τ is compactified with period

$$\beta = \frac{4\pi l^2}{(p+1)r_0}\,. \tag{4.2}$$

We also take the x^i to be compactified on a $p-1$-dimensional torus of volume V_{p-1}. This is a smooth metric, but it has negative energy,

$$E = -\frac{r_0^{p+1}\beta V_{p-1}}{16\pi G_{p+2} l^{p+2}}\,, \tag{4.3}$$

where a definition of E has been used in which the energy of AdS space vanishes.

If one dimensionally reduces along τ, this looks like a $p+1$-dimensional spacetime with a negative-energy "brane" at $r = r_0$. Now, for the metric (4.1) the proper length of the τ circle diverges at large r so that $p+2$-dimensional physics is relevant at any value of β. On the other hand, one can make the physics truly $p+1$-dimensional by cutting of the spacetime at large r as in [33]. A collision with a black hole in the $p+1$-dimensional spacetime is then described by considering a black string wrapping the τ direction.

This example differs from the previous cases in that the amount of negative energy is a free parameter (controlled by β), so that it would be difficult to resolve our concerns using fundamental corrections to the supergravity description. In addition, like the O8-brane, the AdS soliton is co-dimension one when considered as a brane and one may check directly that it is gravitationally attractive.

Remarkably, the brane at $r = r_0$ *cannot* cross the black hole horizon in the usual sense. Since $r = r_0$ is the origin of the r, τ plane in the $p+2$-dimensional geometry, an S^1 of horizon generators would go to zero size if the black string horizon crossed $r = r_0$. Since this geometry is smooth, such a caustic cannot occur on an event horizon [6, 24]. However, the $p+1$-dimensional black hole is assuredly attracted to the negative-energy "brane". So what happens? The only possibility is that the brane enters the horizon at a past endpoint, where new generators are entering the horizon and the black hole is still forming. One can see that this is so by again using the rotational symmetry. The point $r = r_0$ cannot cross the event horizon along a null generator, as there is no distinguished direction for the corresponding null ray to be travelling. Since the negative-tension brane does not intersect a pre-existing family of horizon generators, the argument in the introduction is simply inapplicable. Once again, in the $p+2$-dimensional description, the geometry is everywhere smooth so that the horizon area will not decrease. In particular, the final horizon area must be greater than any reasonable definition of an "initial area."

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Note that in contrast to recent work on higher-dimensional solutions [47], we don't expect the horizon of our $p+2$ "black string" to be toroidal in any invariant sense; the picture here is more like [48], where one forms a horizon with initially toroidal space-like slices, but the hole closes up faster than the speed of light. In the $3+1$ analogue [48], there are no causal curves linking the torus[9] which extend to infinity (in agreement with topological censorship [49]). The cosmological constant (i.e. the r^2/l^2 factor in $g_{\tau\tau}$) will accelerate the wrapped black brane toward $r = r_0$ even if we place the string far away initially, so the "donut hole" in the middle can close up before observers can pass through it.

Finally, we note an interesting property of this "brane" that is intimately associated with the resolution above. Because in $p+2$ dimensions the τ circle shrinks to zero size at $r = r_0$, the $p+1$-dimensional Einstein frame metric necessarily degenerates at the location of the brane. Such a singularity of the $p+1$ metric then forces one to embed the description of this brane in some theory that is more complete than just $p+1$-dimensional Einstein-Hilbert gravity in order to describe the brane. Thus we have once again failed to find a stringy negative tension brane which couples only to Einstein-Hilbert gravity.

5. Discussion

We have addressed a number of stringy negative tension branes and, in each case, we have identified a plausible mechanism that could make these objects compatible with the generalized second law. We have provided various pieces of evidence, including scaling analyses, perturbative treatments, finding non-perturbative corrections to moduli spaces, and analyses of black hole event horizons.

The two cases of the O8-plane and the AdS soliton were exceptional. For the O8-plane we argued that it is only the required proximity to positive tension D8-branes and the distortions these objects induce in nearby black holes that allows the second law to hold. It would clearly be of interest to compute such distortions, perhaps in the perturbative regime of a small black hole, in order to verify our conjecture that the distortions increase the surface area. On the other hand, we argued for the AdS soliton that it was simply impossible for this brane to meet a ten-dimensional event horizon except at the initial events where the horizon first forms and null generators are added.

In most of the orientifold cases, it was possible to identify a set of corrections that should significantly modify the naïve picture of an infinitely thin negative energy brane interacting with simple (Einstein-Hilbert like) gravitational degrees of freedom. In the O6 case we have seen that these corrections have the interpretation of resulting from massive fields for which the orientifold acts as a source, such that the massive fields have order-one excitations near the orientifold. We believe that this picture holds in general, and the instanton corrections identified for the O5-plane are consistent with this conjecture. These massive fields should have curvature couplings to the naïve Einstein metric, so that in

[9] In our higher-dimensional setting this question is moot as tori of our dimension link with surfaces and not curves.

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terms of this metric the entropy of a black hole will not be given exactly by 1/4 of the area in Planck units. Such corrections to the entropy should be relevant when an orientifold encounters the horizon of the black hole.

If the black hole completely engulfs the orientifold then these corrections should die out as the system equilibrates. However, for such cases the orientifold has co-dimension greater than 1 and, as described in the introduction, energy considerations suggest that second law violations are of concern only during the collision and not in the final equilibrium state. It is therefore fitting that our resolution should involve corrections of a similar transitory nature.

When we spoke of corrections above, we generally mean ways in which a spacetime description of an orientifold would differ from that obtained by analytic continuation from the positive tension D-branes to appropriate negative tensions. Although we argued that such corrections should exist, we did not map out these corrections in detail for any case other than the O6-plane. Some progress was made for the O5-plane, where we saw that the corrections were related to the Kosterlitz-Thouless unbinding of instanton/anti-instanton pairs.

It would be interesting to find the explicit form of these corrections in other cases as well. Recall, however, that our interest is generally in the region close to the orientifold where we expect the corrections to dominate over the negative tension D-brane geometry. In contrast, natural methods to compute these effects do so in the regime where the corrections are small. While such calculations are therefore unlikely to produce fully satisfactory results, it may be that they reveal interesting properties as we saw in the perturbative calculations of sections 2.3 and 2.4.

The discussion of energy and entropy above was carried out using the naïve Einstein frame. The reader may wonder if one can do better by correcting the metric to obtain a "true Einstein frame." For definiteness, let us return to the case of the O6-plane and the Atiyah-Hitchin metric in 11-dimensions. In ten dimensions, one may try to find a true Einstein frame by correcting the definition (2.11) of the ten-dimensional Einstein metric at each order in h_{ab}. That this is in fact possible may be seen from the fact that the curvature couplings result from the expansion of the eleven-dimensional Ricci tensor and are thus linear the ten-dimensional Ricci tensor — the terms do not involve higher powers of the Ricci tensor or other components of the ten-dimensional Riemann tensor. Such terms can always be cancelled by the change in the ten-dimensional Einstein term (essentially the Einstein tensor contracted with the modification of $g^{E,10}_{ab}$) induced by an appropriate modification of the metric.

As a result, a true Einstein frame will exist at each perturbative order. Since black hole entropy will be given by the area in this true Einstein frame, this area must match the area as measured by the eleven-dimensional metric. Thus, to the extent that one works in such a true Einstein frame, one expects the description of the physics to match that given in eleven dimensions. One should therefore not find any localized negative energy at all and, to the extent that such a frame makes sense non-perturbatively, the geometry should close off smoothly at the origin with no singular brane to mark any particular points as special. One expects that from this perspective the negative energy is again purely a consequence of the boundary conditions. However, in the weak coupling ($g \to 0$) limit all of this structure shrinks to zero size and one is left with an object effectively described as a negative tension brane with orbifold boundary conditions so long as one does not probe this brane too closely.

Note that the "true Einstein metric" described above involves much more than just a conformal rescaling of the metric. In contrast to the more familiar relation between the string and Einstein frames in massless type-II supergravity, the present change to the true Einstein frame field will therefore introduce a new notion of null rays, causal structure, and black hole horizons.

Interestingly, in no case did we find a resolution in a regime where the negative tension brane could be described in terms of couplings to pure Einstein-Hilbert gravity. For most of the orientifolds, extra massive fields with complicated curvature couplings seemed to be required for thermodynamic stability. While the O8-brane did not require massive fields in quite the same sense, its resolution resulted from the couplings of massive type-IIA supergravity which force the solution to become singular at a finite distance from an isolated brane. When the AdS soliton was viewed as a negative tension brane, the metric necessarily degenerated at the brane and thus passed out of the regime of pure gravity. These are in sharp contrast to the rather benign looking phenomenological negative tension branes used in e.g. [5], for which the interaction of the metric and brane can be described in terms of the Israel junction conditions. It appears that string theory gives no motivation for stability of simple phenomenological negative tension branes coupled to Einstein-Hilbert like gravity.

While we have by no means ruled out further mechanisms to enforce the second law, it is far from clear what these might be. As a result, one suspects that generic phenomenological negative tension branes face difficulties with the second law and, even when placed at an orbifold, are likely to experience instabilities involving black holes of the sort discussed in [26, 28]. While some more complete treatment will therefore eventually be required, the simplicity of such phenomenological negative tension branes will no doubt maintain their important role in exploring novel and interesting new applications of negative energy objects.

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